Engineering Notes

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Crossflow Profiles for Compressible, Turbulent Boundary Layers

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Nomenclature

A = $-(\partial w/U/\partial\zeta)_{\zeta=1}$
B = $(\partial w/U/\partial\zeta)_{\zeta=0}$
u = boundary-layer velocity component parallel to an outer flow streamline
U = magnitude of the outer flow
w = boundary-layer velocity component normal to an outer flow streamline
δ = boundary-layer thickness
ϕ = circumferential angle measured from the windward generator of a yawed cone
ζ = u/U

Introduction

IN a report to the NASA Subcommittee on Fluid Mechanics, Howe[1] discussed several important areas of research related to modern subsonic and supersonic aircraft that require further investigation. It was additionally emphasized that finding solutions to these problems is crucial for continued progress in the aeronautics industry. Several of the problem areas considered are related to the study of compressible, turbulent boundary layers. For example, it was pointed out that for the problem of shock wave impingement on turbulent boundary layers an adequate description of the boundary-layer velocity profiles through the interaction region is not yet available. It was only recently that Mathews et al.[2] showed that a compressible form of the wall-wake profile provides a good representation for the velocity profiles encountered within and downstream of both oblique and normal shock wave boundary-layer interactions.

Another significant area of research, suggested by Howe[1] as requiring further investigation, is the effect of three-dimensionality on compressible, turbulent boundary layers. In particular, the need for a convenient crossflow velocity profile model that allows S-shaped velocity distributions was emphasized. Such profiles have been observed under compressible flow conditions by Hall and Dickens[3] and by Rainbird[4] who measured, respectively, the boundary layers on a side wall of a supersonic nozzle and on a yawed cone. The nozzle flow of Ref. 3 simulated the supersonic flow conditions found on wings with swept leading edges. It is the purpose of this report to show that a family of hodograph models, previously applied to the crossflow velocity component of three-dimensional, turbulent, incompressible boundary layers, provides an adequate representation for compressible crossflow profiles with (S-shaped profiles) and without (D-shaped profiles) flow reversal.

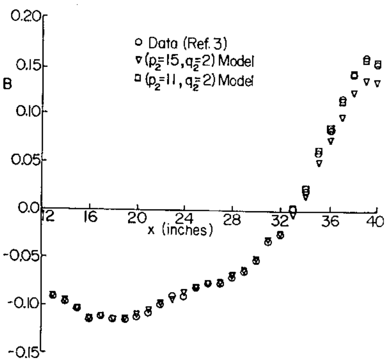

Fig. 1 Comparison of *B* values with Hall and Dickens' data.

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(p_i,q_j) Family of Crossflow Models

Shanebrook and Hatch[5,6] have presented a family of crossflow models that is based on a polynomial representation for the hodograph of w/U vs u/U. The principal advantage of this family is its flexibility which derives mainly from its ability to control the extent of experimentally observed linear regions as the wall and boundary-layer edge are approached. That is, assuming $w/U(\zeta)$ and writing a Taylor series expansion about $\zeta = 0$,

$$\frac{w}{U} = B\zeta + \left(\frac{\partial^2 w/U}{\partial\zeta^2}\right)_{\zeta=0}\cdot\frac{\zeta^2}{2!} + \left(\frac{\partial^3 w/U}{\partial\zeta^3}\right)_{\zeta=0}\cdot\frac{\zeta^3}{3!} + \ldots$$

and another about $\zeta = 1$,

$$\frac{w}{U} = -A(\zeta - 1) + \left(\frac{\partial^2 w/U}{\partial\zeta^2}\right)_{\zeta=1}\cdot\frac{(\zeta-1)^2}{2!} + \left(\frac{\partial^3 w/U}{\partial\zeta^3}\right)_{\zeta=1}\cdot\frac{(\zeta-1)^3}{3!} + \ldots$$

it can be seen that the observed linear variations can be mathematically approximated by requiring one or more consecutive higher-order derivatives to be zero at the wall and/or boundary-layer edge. By varying the number of consecutive higher-order zero derivatives it is possible to generate a wide variety of possible shapes for the hodograph since in this way the extent of linearity can be controlled.

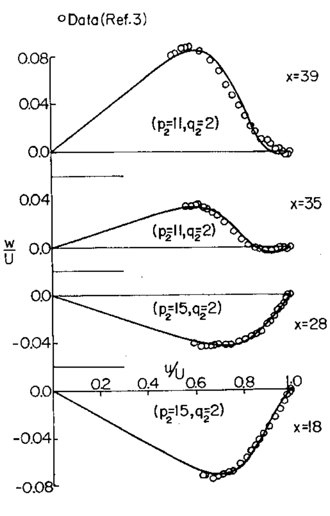

Fig. 2 Hodograph comparisons for data of Hall and Dickens.

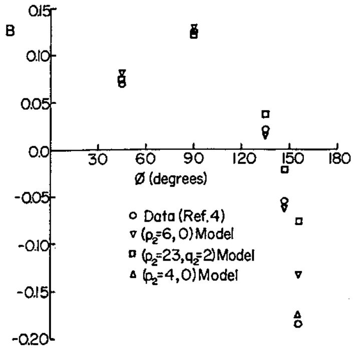

Fig. 3 Comparison of B values with Rainbird's data.

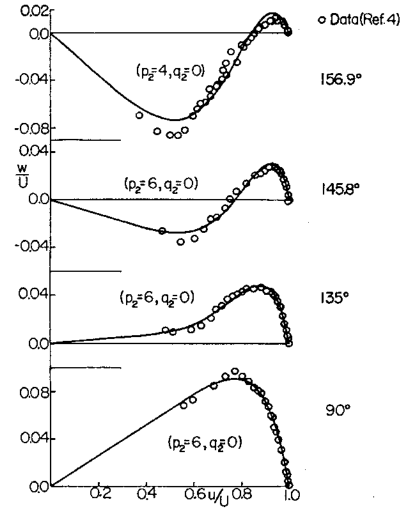

Fig. 4 Hodograph comparisons for data of Rainbird.

The remaining conditions adopted for the polynomial representation are presented in Refs. 5 and 6. The resulting family has been termed the (p_i,q_j) family of hodograph models where p_i is the number of zero derivatives at the wall beginning with the ith derivative and q_j is the number of zero derivatives at the boundary-layer edge beginning with the jth derivative. The subscripts i and j must be greater than unity and for the special case where no higher-order zero derivatives are specified at the wall or boundary-layer edge a zero is substituted for p_i or q_j, respectively. Algebraic forms expressing $w/U = f(\zeta,A,B,p_2)$ for the subfamilies $(p_2,0)$, $(p_2,q_2 = 1)$, and $(p_2,q_2 = 2)$ are given in Refs. 5 and 6.

Comparisons of selected members of the (p_i,q_j) family with experimental hodographs with and without flow reversal, obtained under incompressible flow conditions, have been made in Ref. 5. A momentum integral method, based on this family of crossflow models, for three-dimensional, turbulent, incompressible boundary layers, was presented in Ref. 6 where it was shown that successful calculations can be performed in regions of S-shaped as well as D-shaped crossflow profiles.

Comparisons with Compressible Hodograph Data

As mentioned previously, Refs. 3 and 4 present compressible crossflow profile data for turbulent boundary-layer flow over a nozzle wall and a yawed cone, respectively. Both references compare their D- and S-shaped profiles with Mager's[7] crossflow model,

$$w/U = B(u/U)[1 - (y/\delta)]^2 \tag{1}$$

Unfortunately, this simple model does not provide an adequate representation for compressible crossflow profiles although it is applicable to certain incompressible flowfields.

To illustrate the applicability of the (p_i,q_j) family for representing compressible crossflow profiles, the method of least squares was used to fit several of the experimental hodographs from Refs. 3 and 4 with crossflow models from the $(p_2,0)$, $(p_2,q_2 = 1)$, and $(p_2,q_2 = 2)$ subfamilies. Figure 1 compares values of the parameter B determined from the least squares fit with experimental values measured by Hall and Dickens[3] along their B streamline. The $(p_2 = 15,\ q_2 = 2)$ model provides excellent agreement with the data until B passes through zero ($x \cong 33$ in.) after which it somewhat underpredicts the B values. The crossflow profiles immediately downstream of the station where B is zero are S shaped and further yet downstream become D shaped again, indicating complete reversal of the crossflow. It was found that the $(p_2 = 11,\ q_2 = 2)$ model gives better B values for the S- and D-shaped crossflow profiles downstream of the station where B passes through zero as shown in Fig. 1. Crossflow models from the $(p_2,0)$ and $(p_2,q_2 = 1)$ subfamilies also gave results for the parameter B comparable to those shown in Fig. 1. However, the best least squares fit of the experimental hodographs was obtained with crossflow models from the $(p_2, q_2 = 2)$ subfamily and representative comparisons are shown in Fig. 2.

Figure 3 compares values of the parameter B determined from the least squares fit with experimental values measured by Rainbird[4] on a yawed cone at an angle of attack of 15.7° and a freestream Mach number of 1.8. The $(p_2 = 23,\ q_2 = 2)$ model is satisfactory until B passes through zero. However, the $(p_2 = 6,\ 0)$ model is very good until the last station shown where the $(p_2 = 4,\ 0)$ model gives a better value for the parameter B. It should be noted that this station is very close to the observed separation line at $\phi = 159°$. Figure 4 compares the experimental hodographs with the least squares fit obtained using, where appropriate, the $(p_2 = 6,0)$ and $(p_2 = 4,0)$ models.

The method of least squares was also used to fit Eq. (1) to the aforementioned experimental hodographs from Refs. 3 and 4. However, the resulting B values were generally poor and the hodograph comparisons were similar in quality to the profile comparisons shown in Refs. 3 and 4.

Conclusions

Based on the results shown in Figs. 1–4 it may be concluded that the (p_i,q_j) family of hodograph models provides an adequate representation of the existing experimental D- and S-shaped compressible crossflow profiles. It is therefore recommended that this family of crossflow models be considered in future developments of practical momentum integral methods for predicting the behavior of three-dimensional, compressible, turbulent boundary layers.

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